General Solution Example 1

Example 6: Consider the 3×4 linear system with variables x_1, x_2, x_3, x_4 that has the given corresponding augmented matrix. Write the augmented matrix in *reduced row echelon form.* Find the general solution of the linear system in *parametric form.*

$$
\begin{array}{cccc}
x_1 & x_2 & x_3 & x_4
\end{array}
$$

$$
\left[\begin{array}{cccc|c}
0 & 2 & 0 & 2 & 2 \\
1 & 2 & -1 & 3 & 5 \\
2 & 4 & -1 & 6 & 3
\end{array}\right] \tag{1}
$$

Second step ↓

$$
\left[\begin{array}{cccc|c}
0 & 2 & 0 & 2 & 2 \\
1 & 2 & -1 & 3 & 5 \\
2 & 4 & -1 & 6 & 3
\end{array}\right]
\begin{array}{l} R_3 := R_3 - 2R_1 \\ R_1 \Leftrightarrow R_2 \end{array}
\left[\begin{array}{cccc|c}
1 & 2 & -1 & 3 & 5 \\
0 & 2 & 0 & 2 & 2 \\
0 & 0 & 1 & 0 & -7
\end{array}\right]
$$

First step

bound variables:
x_1, x_2, x_3

free variables:
x_4

$$
R_1 := R_1 + R_3
\left[\begin{array}{cccc|c}
1 & 2 & 0 & 3 & -2 \\
0 & ② & 0 & 2 & 2 \\
0 & 0 & 1 & 0 & -7
\end{array}\right]
\begin{array}{l} R_2 := \frac{1}{2} R_2 \\ R_1 := R_1 - R_2 \end{array}
\left[\begin{array}{cccc|c}
1 & 0 & 0 & 1 & -4 \\
0 & 1 & 0 & 1 & 1 \\
0 & 0 & 1 & 0 & -7
\end{array}\right]
\qquad x_4 = t
$$

$$
\begin{cases}
x_1 + x_4 = -4 \\
x_2 + x_4 = 1 \\
\quad x_3 = -7
\end{cases}
\Rightarrow
\begin{array}{l}
x_1 + t = -4 \\
x_2 + t = 1
\end{array}
\Rightarrow
\begin{array}{l}
x_1 = -4 - t \\
x_2 = 1 - t
\end{array}
$$

$$
\boxed{
\begin{array}{l}
x_1 = -4 - t \\
x_2 = 1 - t \\
x_3 = -7 \\
x_4 = t
\end{array}
}
$$